Quantum Laboratory Notebook

Chapter 2 — A Three-Qubit GHZ / Mermin Test on Real Quantum Hardware

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Project type: Independent personal research and educational project
Series: Quantum Laboratory Notebook — Chapter 2 (companion to the Bell–CHSH notebook)
Platform used: Quantum Inspire — Tuna-17 backend
Language: cQASM 3.0
Date of experiment: 19 July 2026
Session: 18:22 – 18:36 (five runs)

Notebook entry — 19 July 2026

Yesterday I violated the CHSH inequality with two entangled qubits. Today I went one qubit further. I prepared a three-qubit **GHZ state** — the same recipe as the Bell state, with one extra CNOT — and measured it in the settings of the **Mermin** argument. With three qubits the clash with local realism becomes sharper: it no longer hides inside a statistical average, it shows up directly in the *signs* of the correlations. This notebook records the five programs I ran, the raw data the machine returned, the analysis, and one honest surprise about a sign convention that only a real device would have taught me.

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1. Abstract

In 1989 Greenberger, Horne and Zeilinger showed that three entangled particles expose the conflict between quantum mechanics and local realism far more sharply than two do. Where Bell's two-particle argument produces a *statistical* inequality — you have to average many runs to see the violation — the three-particle GHZ argument produces an *all-or-nothing* contradiction: for four specific measurement settings, local realism and quantum mechanics predict opposite signs for the same quantity. Mermin turned this into a single number M that any local-realistic theory must keep at **$|M| \le 2$**, while quantum mechanics allows it to reach **$|M| = 4$**.

In this experiment I prepared the three-qubit GHZ state

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

on the Tuna-17 quantum processor, measured it in the four Mermin settings (plus one structural check), and combined the four correlations into M. The experiment returned

$$|M| = 3.417 \pm 0.011.$$

This is far above the classical limit of 2 and below the quantum maximum of 4, exactly as quantum mechanics predicts for a real, slightly noisy device. The classical bound is exceeded by about **123 standard deviations** — a much larger margin than the 34σ of the two-qubit CHSH test on the same machine, which is precisely the point of going to three qubits.

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2. Objective

The goal is a direct continuation of Chapter 1:

> **Prepare a three-qubit GHZ state on real hardware, measure the Mermin quantity *M*, and check whether it exceeds the local-realistic limit of 2.**

A second, quieter question rides along with it: **does adding a third qubit and a second entangling gate destroy the state?** On present-day hardware every extra gate is an extra chance to introduce noise, so it is not obvious in advance that a three-qubit entangled state survives well enough to violate anything. Measuring the fidelity and visibility of the prepared state answers that question with numbers.

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3. Theory

3.1 The GHZ state

The experiment begins by preparing three qubits in the entangled GHZ state

$$|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$

This is the natural big brother of the Bell state. The Bell state $|\Phi^+\rangle$ is made with a Hadamard followed by one CNOT; the GHZ state uses **exactly the same recipe with one more CNOT**, copying the correlation onto a third qubit:

```
H     q[0]          // put qubit 0 into an equal superposition
CNOT  q[0], q[2]     // entangle qubit 2 with it
CNOT  q[0], q[1]     // entangle qubit 1 with it
```

In this state each qubit on its own looks completely random, but all three are locked together: measured in the computational basis, the only outcomes that ever appear are **000** and **111**, each with probability ½. There is no half-way house.

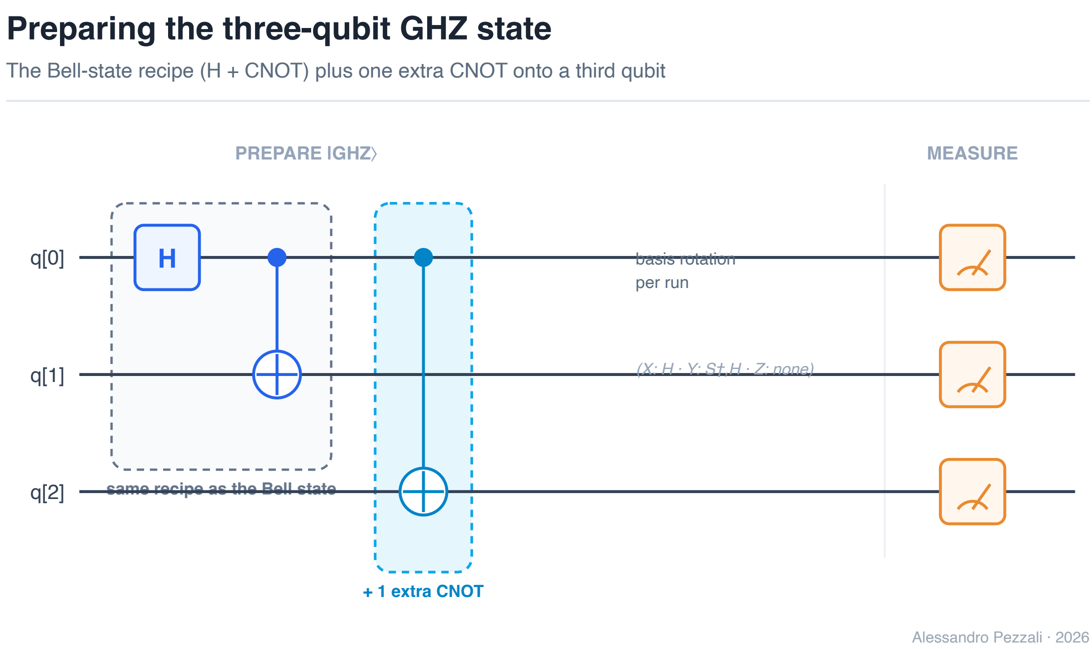

Figure 1. The GHZ preparation circuit. The first two gates (**H** then **CNOT**) are the *same* block that prepared the Bell state in Chapter 1; the single extra **CNOT** onto qubit 2 is the entire difference between a two-qubit and a three-qubit entangled state.

3.2 Why three qubits are special — the Mermin argument

To test local realism we measure each qubit in either the **X** or the **Y** basis, in four carefully chosen combinations:

$$XXX, \qquad XYY, \qquad YXY, \qquad YYX.$$

For each setting the machine returns a string of three bits; we assign it a correlation value by the **parity rule** — outcomes with an *even* number of 1s count as +1, outcomes with an *odd* number of 1s count as −1 — and average over the shots to get the expectation values ⟨XXX⟩, ⟨XYY⟩, ⟨YXY⟩ and ⟨YYX⟩.

Here is the crux, and it needs *no* averaging to state. Suppose the world were locally real: every qubit i carries predetermined answers $m_X^i,\ m_Y^i \in \{+1,-1\}$ that it would give to an X or a Y measurement, fixed before we choose which to perform. Then the three Y-containing products are forced to satisfy an exact identity, because every Y value appears **twice** and $(m_Y^i)^2 = 1$:

$$(XYY)(YXY)(YYX) \;=\; m_X^1 m_X^2 m_X^3\,(m_Y^1 m_Y^2 m_Y^3)^2 \;=\; XXX.$$

So *any* local-realistic model must obey

$$(XYY)(YXY)(YYX) \;=\; XXX.$$

Quantum mechanics flatly refuses. For the GHZ state the three mixed settings are perfectly correlated with one sign and XXX with the opposite sign, so the left-hand side and the right-hand side come out **opposite**: a bare contradiction, $+1 \neq -1$, with no statistics anywhere in sight. A single perfect shot of each setting would already settle it. That is the sense in which three qubits are sharper than two — Bell's two-qubit inequality is a statement about averages, GHZ is a statement about individual signs.

3.3 The Mermin quantity

Real devices are noisy, so instead of demanding perfect ±1 values we use Mermin's robust, statistical version. Define

$$M \;=\; \langle XYY\rangle + \langle YXY\rangle + \langle YYX\rangle - \langle XXX\rangle.$$

Any local-realistic theory obeys **$|M| \le 2$**. Quantum mechanics allows M to reach **4**, and the GHZ state saturates it: up to the overall convention fixed by how the measurement outcomes are labelled, the ideal predictions are

$$\langle XXX\rangle = -1, \qquad \langle XYY\rangle = \langle YXY\rangle = \langle YYX\rangle = +1,$$

— **three correlations of one sign and a single one of the other** — so that ideally $M = (+1) + (+1) + (+1) - (-1) = \mathbf{4}$. It is this "three-and-one" sign structure, not the individual signs themselves, that carries the physics; the overall sign is a convention, a point that will matter when we look at the data (Section 7.4).

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4. Experimental Design

Why these five particular measurements?

The Mermin quantity is built from four correlations, so the test needs **four runs** — one each for XXX, XYY, YXY and YYX. I added a **fifth run, ZZZ**, not to feed M but as a structural check: measured in the computational basis, a good GHZ state must show only **000** and **111**. That fifth run measures how well the state was actually prepared before any clever basis rotation, and gives the fidelity quoted later.

Each run uses the *same* GHZ preparation; only the final single-qubit basis rotations change. This keeps the comparison clean — any difference between runs comes from the measurement setting, not from a different starting state. The basis rotations are the textbook ones:

Basis	Measures along	Realised in the circuit by
Z	Z axis	measure directly (no rotation)
X	X axis	apply **H** before measuring
Y	Y axis	apply **S†** then **H** before measuring

Table 1. The three measurement bases and how each is realised on the hardware.

Crossing these bases in the Mermin pattern gives the five configurations:

Run	Setting	Qubit 0	Qubit 1	Qubit 2	Purpose
GHZ_0	Z Z Z	Z	Z	Z	structural check — fidelity of $\|000\rangle+\|111\rangle$
GHZ_1	X X X	X	X	X	correlation $\langle XXX\rangle$
GHZ_2	X Y Y	X	Y	Y	correlation $\langle XYY\rangle$
GHZ_3	Y X Y	Y	X	Y	correlation $\langle YXY\rangle$
GHZ_4	Y Y X	Y	Y	X	correlation $\langle YYX\rangle$

Table 2. The five measurement configurations. Runs 1–4 supply the Mermin correlations; run 0 verifies the prepared state.

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5. Methods

Hardware and software. All five programs were written in cQASM 3.0 and executed on the Quantum Inspire platform using the **Tuna-17** backend. Each program used three qubits and three classical bits.

State preparation. Every run starts with the same three gates:

```
H    q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]
```

The Hadamard puts qubit 0 into an equal superposition; the two CNOTs then entangle qubits 2 and 1 with it, producing the GHZ state $|000\rangle + |111\rangle$ (up to normalisation).

Measurement settings. After preparation, each run applies the single-qubit basis rotations for its configuration (Section 4) and then measures all three qubits:

```
b = measure q
```

Statistics. Each configuration was executed with **8192 shots**. The platform reports the outcome probabilities for the eight results 000 … 111.

Session log. The five runs were executed one after another in a single session:

Run	Setting	Job ID	Result ID	Timestamp	Shots	Program hash
GHZ_0	Z Z Z	1151305	1116830	19/07/2026 18:22:50	8192	1165218@0e697d6c
GHZ_1	X X X	1151329	1116854	19/07/2026 18:25:32	8192	1165245@0648f66e
GHZ_2	X Y Y	1151349	1116874	19/07/2026 18:27:50	8192	1165264@f11d2c06
GHZ_3	Y X Y	1151376	1116905	19/07/2026 18:30:40	8192	1165292@0e9fc1d0
GHZ_4	Y Y X	1151394	1116926	19/07/2026 18:36:28	8192	1165313@ab624825

Table 3. Session log for the five runs, as reported by the platform.

The original platform screenshots for all five runs are reproduced in Appendix B.

6. Raw Data

The platform returned the outcome probabilities below (8192 shots per run).

Run	Setting	P(000)	P(001)	P(010)	P(011)	P(100)	P(101)	P(110)	P(111)
GHZ_0	Z Z Z	0.465	0.004	0.019	0.009	0.009	0.013	0.008	0.473
GHZ_1	X X X	0.238	0.014	0.015	0.225	0.017	0.241	0.232	0.017
GHZ_2	X Y Y	0.023	0.217	0.218	0.021	0.233	0.021	0.027	0.241
GHZ_3	Y X Y	0.020	0.226	0.213	0.016	0.239	0.019	0.019	0.247
GHZ_4	Y Y X	0.016	0.223	0.246	0.015	0.233	0.016	0.016	0.237

Table 4. Measured outcome probabilities for the five runs.

Two features are already visible by eye. In **GHZ_0** (Z Z Z) essentially all the weight sits on **000** (0.465) and **111** (0.473); the other six outcomes together carry only about 6 %. That is the GHZ signature — the state was prepared well. In the four measurement runs the weight instead spreads across the four bitstrings of a single parity, which is exactly what produces a large correlation of definite sign.

Figure 2. The measured probability distributions for the five runs. GHZ_0 shows the two “all-agree” outcomes 000 and 111 towering over the rest. In GHZ_1 the even-parity outcomes (000, 011, 101, 110) dominate; in GHZ_2, GHZ_3 and GHZ_4 the odd-parity outcomes (001, 010, 100, 111) dominate instead — the sign flip that drives the Mermin violation.

7. Analysis

7.1 The parity rule

For every run the correlation is the parity-weighted sum of the eight probabilities,

$$E = \sum_{\text{even } s} P(s) - \sum_{\text{odd } s} P(s),$$

where *even* means an even number of 1s (000, 011, 101, 110) and *odd* means an odd number (001, 010, 100, 111). The value runs from +1 (parity always even) to −1 (parity always odd).

7.2 The four correlations

Applying the parity rule to the four measurement runs:

Run	Setting	Even sum − Odd sum	Correlation
GHZ_1	⟨XXX⟩	0.936 − 0.063	**+0.873**
GHZ_2	⟨XYY⟩	0.092 − 0.909	**−0.817**
GHZ_3	⟨YXY⟩	0.074 − 0.925	**−0.851**
GHZ_4	⟨YYX⟩	0.063 − 0.939	**−0.876**

Table 5. The four measured Mermin correlations, computed from the raw probabilities.

The structural run confirms the state: applying the same rule to GHZ_0 gives ⟨ZZZ⟩ = 0.495 − 0.505 ≈ **−0.01**, i.e. essentially zero, because the GHZ state populates one even outcome (000) and one odd outcome (111) equally — exactly as expected, and not a term that enters M.

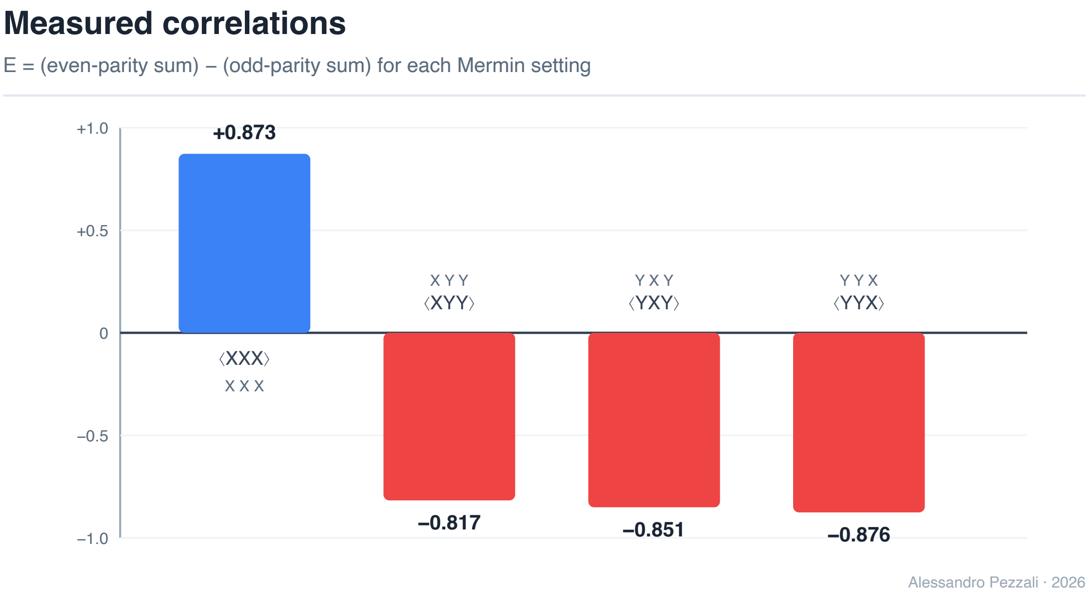

Figure 3. The four measured Mermin correlations. One is strongly positive and three are strongly negative — the "three-and-one" sign structure that the Mermin combination is built to detect.

7.3 The Mermin quantity

Combining the four correlations,

$$M = \langle XYY\rangle + \langle YXY\rangle + \langle YYX\rangle - \langle XXX\rangle$$

$$M = (-0.817) + (-0.851) + (-0.876) - (+0.873) = -3.417,$$

so the measured magnitude is

$$|M| = 3.417.$$

7.4 An honest note on the signs

Note — a fixed sign convention, documented not corrected.

Every one of the four measured correlations came out with the **opposite sign** to the naive "by-hand" prediction of Section 3.3 (which had ⟨XXX⟩ negative and the three mixed settings positive). The whole pattern is globally flipped: here ⟨XXX⟩ is positive and the three mixed settings are negative, so the combination lands at M = **–3.417** rather than +3.417.

This is **not** an error and **not** noise. It is a fixed sign convention in the way the platform implements the **Y-basis measurement** (the $S^\dagger \cdot H$ rotation and the labelling of its outcome). The flip is identical and coherent across all four runs and across the whole session — a random fault could never be that consistent. Crucially, the **Mermin structure is untouched**: still three correlations of one sign and one of the other, and the magnitude **|M| = 3.417** is exactly what it would be either way. Because M enters the test only through |M|, the result does not depend on the convention at all.

I am recording this openly rather than silently absorbing the minus sign, because it is exactly the kind of detail that never appears in a textbook and only surfaces when you run the circuit on real hardware and read the numbers the machine actually gives back.

7.5 Uncertainty

Each correlation is estimated from 8192 shots. The statistical uncertainty of a single correlation is approximately $\sigma_E = \sqrt{(1-E^2)/N}$, giving between ±0.005 and ±0.006 per run. Combining the four in quadrature:

$$\sigma_M = \sqrt{\sigma_{XXX}^2 + \sigma_{XYY}^2 + \sigma_{YXY}^2 + \sigma_{YYX}^2} \approx 0.011.$$

So the final result is

$$\boxed{|M| = 3.417 \pm 0.011.}$$

8. Results

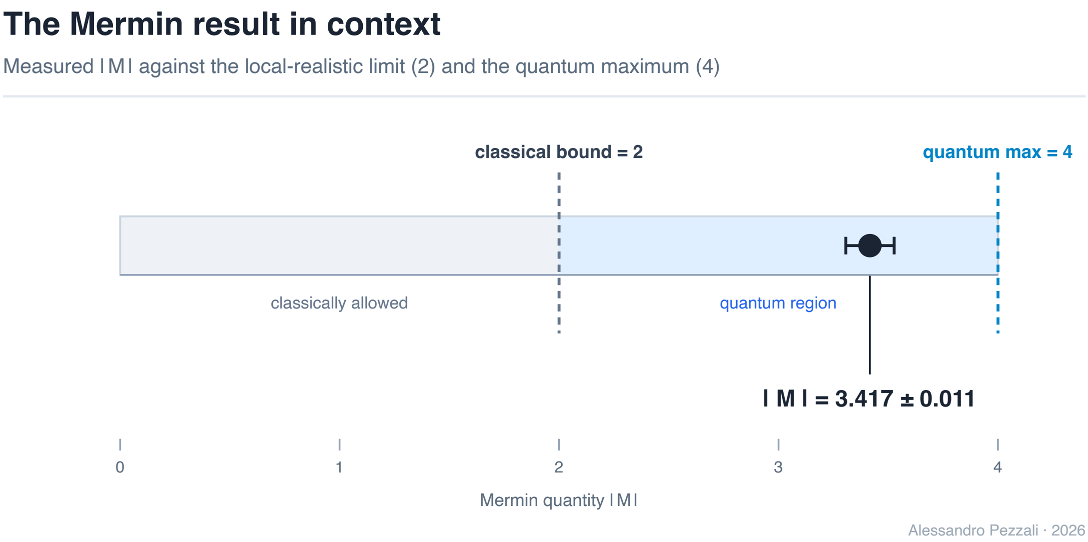

Figure 4. The measured value sits deep inside the quantum region — well above the classical bound of 2 and below the algebraic maximum of 4.

The measured value exceeds the classical limit by

$$\frac{3.417 - 2}{0.011} \approx 123 \text{ standard deviations.}$$

A local-realistic explanation of these runs is therefore excluded with an enormous margin — roughly **3.6 times** the 34σ margin of the two-qubit CHSH test on the same machine. At the same time $|M|$ stays below the quantum maximum of 4, exactly as expected from a real device.

Two further numbers summarise the quality of the prepared state:

Quantity	Symbol	Measured	Ideal	Interpretation
Mermin quantity	$\|M\|$	3.417 ± 0.011	4	violation of local realism
State fidelity	P(000) + P(111)	0.938	1	weight on the GHZ outcomes (run ZZZ)
Mean visibility	$\langle\|E\|\rangle = \|M\|/4$	0.854	1	average correlation strength
Margin over bound	$(\|M\|-2)/\sigma$	≈ 123σ	—	statistical significance

Table 6. Summary of results.

9. Discussion

The result behaves as quantum mechanics predicts, and it does so more emphatically than the two-qubit test. Three points are worth drawing out.

1. The violation is enormous, and that is the whole point of GHZ. The classical bound is exceeded by about 123 standard deviations, against 34σ for the CHSH test on the same hardware. This is not because the GHZ state is "more entangled" in some vague sense; it is because the three-qubit argument concentrates the conflict into the *signs* of the correlations rather than into a statistical average, so even a partly degraded state lands far from the classical limit. The mean visibility of 0.854 would already be enough: a perfectly local-realistic source would need $|M| \le 2$, and $0.854 \times 4 = 3.417$ sails past it.

2. Adding a third qubit cost very little. The prepared state has fidelity P(000) + P(111) = **0.938**, and the four correlations have an average magnitude of **0.854**. The corresponding number for the two-qubit Bell state on this same backend was a visibility of about 0.91. So going from two qubits to three — one extra CNOT, one extra qubit's worth of decoherence and readout error — lowered the correlation strength only modestly. The second entangling gate did **not** collapse the state, which was not a foregone conclusion on present-day hardware and is a genuine (if quiet) result of this run.

3. The sign surprise. As documented in Section 7.4, all four correlations arrived with the opposite sign to the by-hand expectation, a fixed convention of the platform's Y-basis measurement. It changes nothing about $|M|$ or the violation, but it is the honest fingerprint of a real device and is recorded here rather than quietly corrected.

Consistency check. The three "mixed" correlations (−0.817, −0.851, −0.876) are close to one another, and $\langle XXX \rangle$ (+0.873) is their mirror in sign and comparable in magnitude — the clean, symmetric "three-and-one" pattern the ideal experiment predicts. Random noise does not produce that structure.

Scope and limits. This notebook documents five runs from a single session. Like the CHSH notebook, it does not close the loopholes of a rigorous multipartite Bell test (locality, detection), and no device-calibration data beyond the platform metadata were recorded. The result should be read as a faithful, reproducible demonstration of a GHZ/Mermin violation on this hardware, not as a loophole-free test.

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10. Conclusion

I prepared a three-qubit GHZ state on the Tuna-17 quantum processor — the same H + CNOT recipe as the Bell state, with one extra CNOT — and measured it in the four Mermin settings. Combining the four correlations gave

$$|M| = 3.417 \pm 0.011,$$

which exceeds the local-realistic limit of 2 by about 123 standard deviations and stays below the quantum maximum of 4. The prepared state had fidelity 0.938 and mean visibility 0.854, showing that the extra qubit and entangling gate cost little coherence. Together with Chapter 1, this notebook demonstrates that the correlations produced by this hardware — for two *and* for three entangled qubits

— cannot be explained by local realism, and that the three-qubit test exposes that fact far more sharply than the two-qubit one.

Appendix A — Source Code

All five programs are written in cQASM 3.0. The preparation block is identical in every run; only the measurement stage changes. A **Z** measurement needs no rotation, an **X** measurement is realised by an **H** before the measurement, and a **Y** measurement by **S†** followed by **H**. The code is reproduced exactly as executed.

A.1 GHZ_0 — Z Z Z (structural check)

```
version 3.0


qubit[3] q
bit[3] b

// Prepare the GHZ state |000> + |111>
H q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]

// Measure all three qubits directly in the Z basis
b = measure q
```

A.2 GHZ_1 — X X X

```
version 3.0

qubit[3] q
bit[3] b

H q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]

// Measure every qubit in the X basis
H q[0]
H q[1]
H q[2]

b = measure q
```

A.3 GHZ_2 — X Y Y

```
version 3.0

qubit[3] q
bit[3] b

H q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]

// q0 in the X basis
H q[0]
// q1 in the Y basis
Sdag q[1]
H q[1]
// q2 in the Y basis
Sdag q[2]
H q[2]

b = measure q
```

A.4 GHZ_3 — Y X Y

```
version 3.0

qubit[3] q
bit[3] b

H q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]

// q0 in the Y basis
Sdag q[0]
H q[0]
// q1 in the X basis
H q[1]
// q2 in the Y basis
Sdag q[2]
H q[2]

b = measure q
```

A.5 GHZ_4 — Y Y X

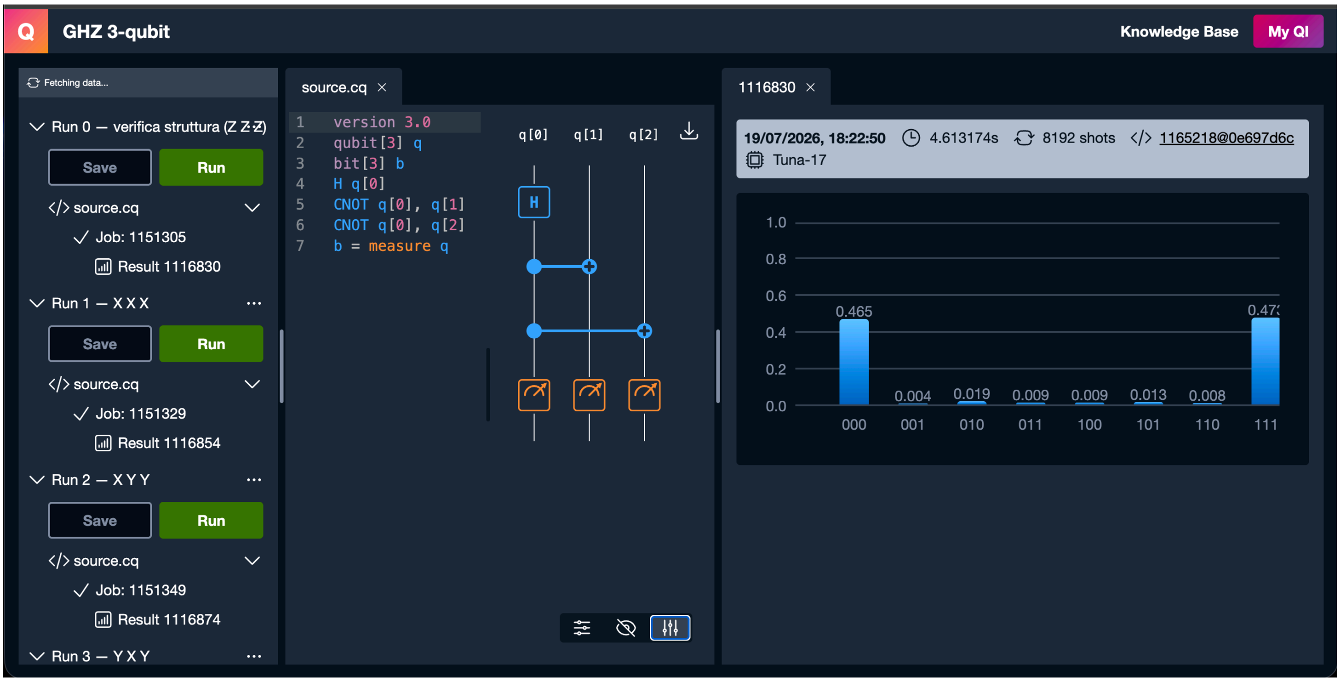
```
version 3.0

qubit[3] q
bit[3] b

H q[0]
CNOT q[0], q[2]
CNOT q[0], q[1]

// q0 in the Y basis
Sdag q[0]
H q[0]
// q1 in the Y basis
Sdag q[1]
H q[1]
// q2 in the X basis
H q[2]

b = measure q
```

> Note: a Y-basis measurement is realised as **S†** then **H** before a Z measurement. The fixed sign convention of this rotation — and of how its outcome is labelled — is the origin of the global sign discussed in Section 7.4.

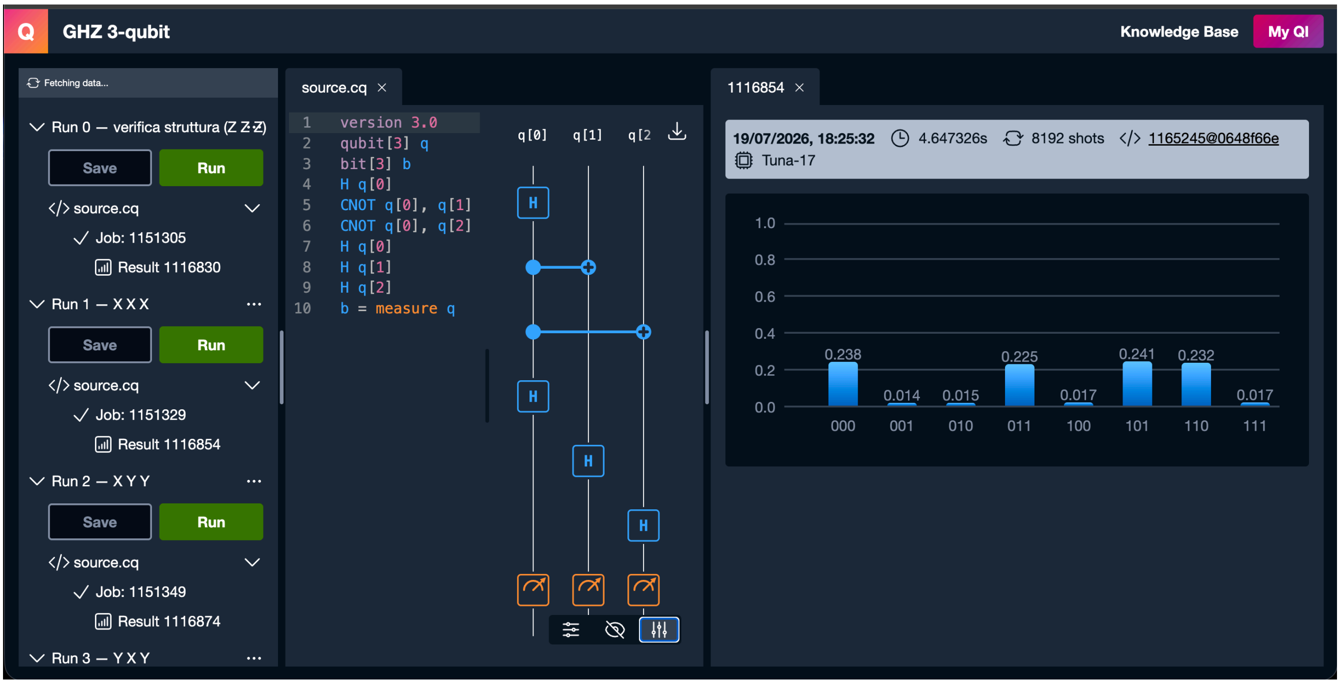
Appendix B — Original Platform Screenshots

The original Quantum Inspire result screens for the five runs, kept for provenance. Each shows the executed program, the circuit, the backend (Tuna-17), the shot count (8192), the timestamp, and the measured outcome probabilities.

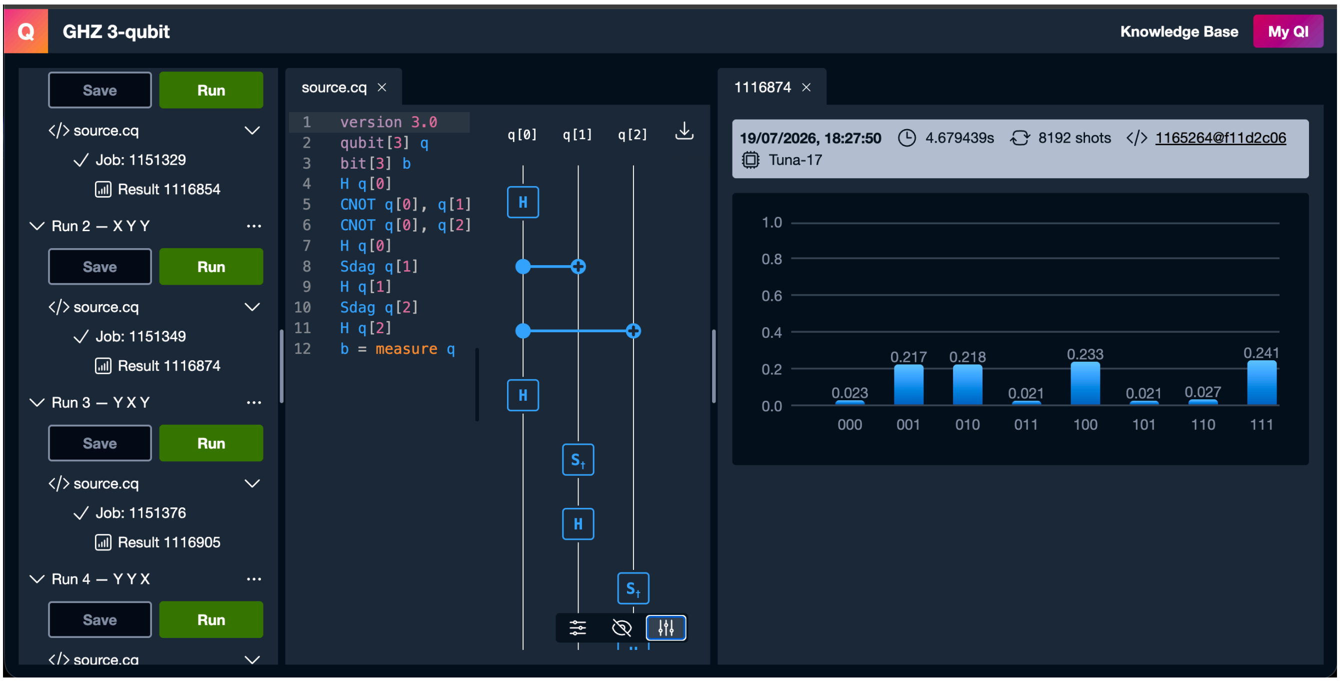
GHZ_0 — Z Z Z — Result 1116830



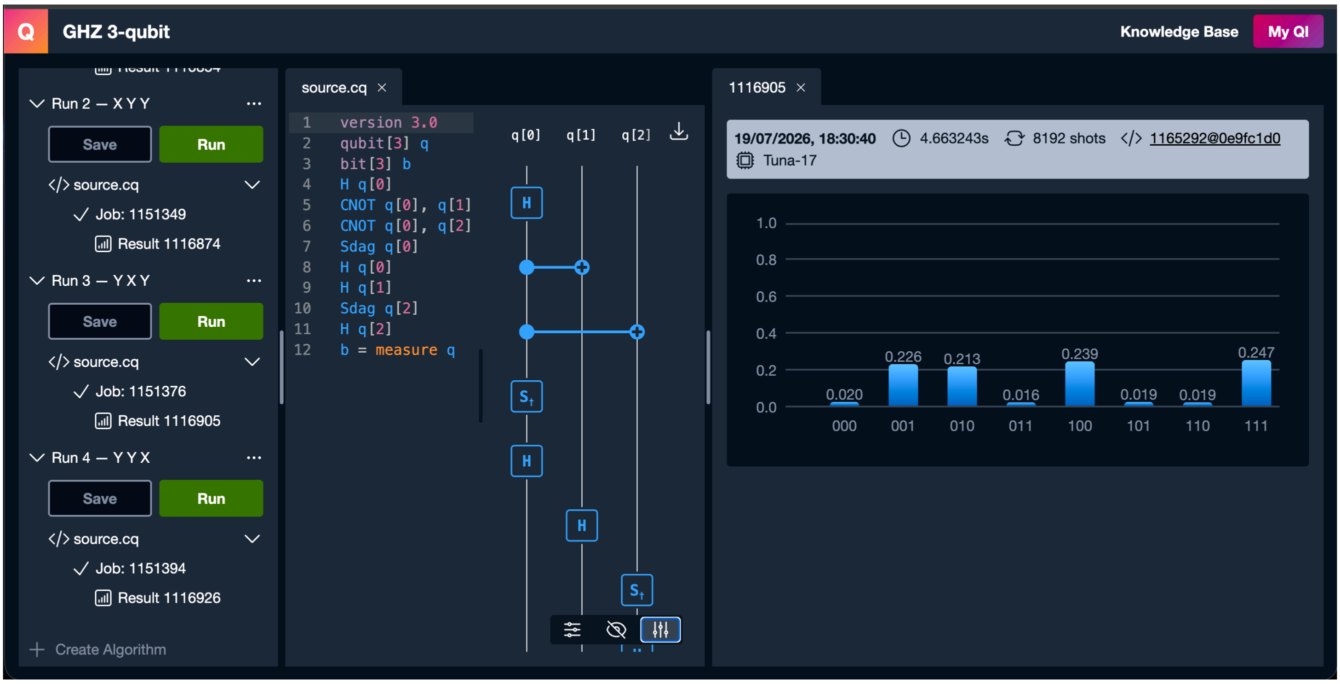
GHZ_1 — X X X — Result 1116854



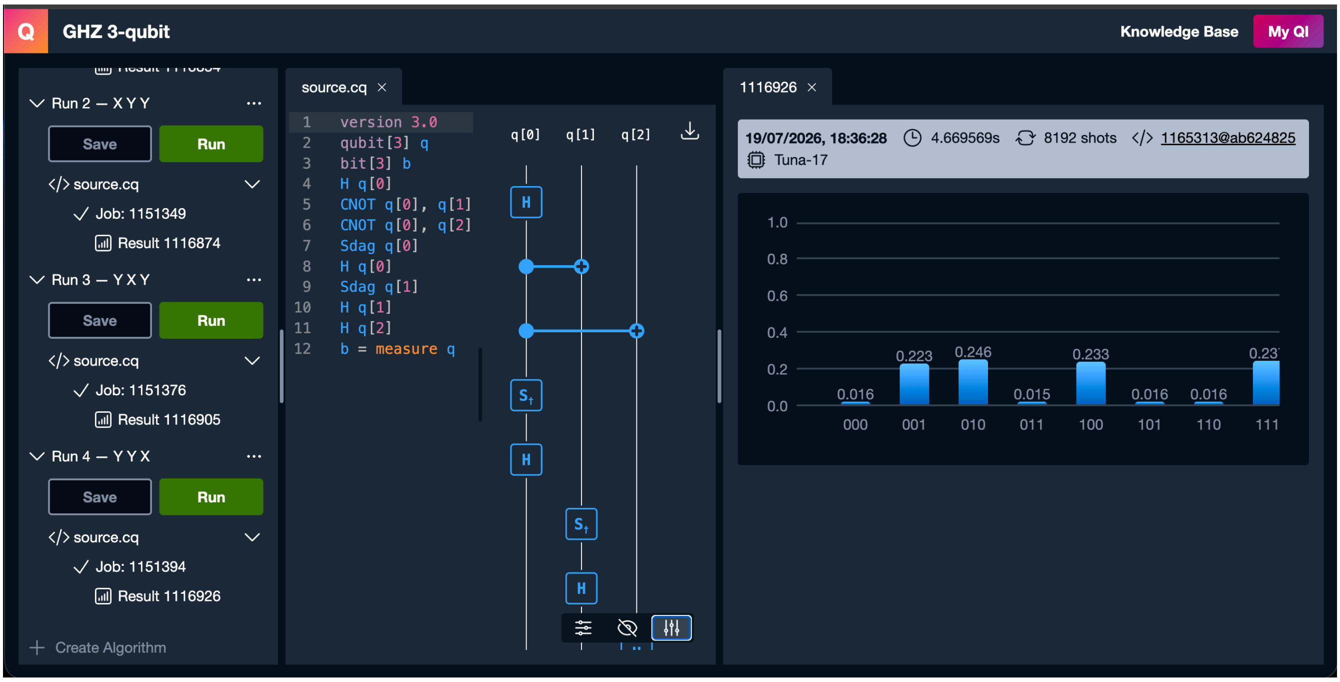
GHZ_2 — X Y Y — Result 1116874



GHZ_3 — Y X Y — Result 1116905



GHZ_4 — Y Y X — Result 1116926



References

1. D. M. Greenberger, M. A. Horne, A. Zeilinger, *Going beyond Bell's theorem*, in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* (ed. M. Kafatos), Kluwer Academic, Dordrecht (1989), p. 69.
2. N. D. Mermin, *Extreme quantum entanglement in a superposition of macroscopically distinct states*, Physical Review Letters **65**, 1838 (1990).
3. Quantum Inspire — quantum computing platform documentation (Tuna-17 backend).
4. A. Pezzali, *Quantum Laboratory Notebook — Testing a Bell–CHSH Inequality on Real Quantum Hardware* (Chapter 1, companion notebook, 18 July 2026).

Sole author: Alessandro Pezzali. Independent personal research and educational project. Quantum Inspire and the Tuna-17 backend are used solely as the experimental platform on which the quantum circuits were executed.