Quantum Laboratory Notebook

Testing a Bell–CHSH Inequality on Real Quantum Hardware

Author: Alessandro Pezzali
Project type: Independent personal research and educational project
Platform used: Quantum Inspire — Tuna-17 backend
Language: cQASM 3.0
Date of experiment: 18 July 2026
Session: 17:28 – 17:31 (four runs)

> **Notebook entry — 18 July 2026**
>
> Today I ran a complete Bell–CHSH test on real quantum hardware. I prepared an entangled pair of qubits, measured it in four different combinations of settings, and used the results to evaluate the CHSH quantity S. This notebook records what I did, the exact programs I executed, the data the machine returned, and how I analysed it. Every number below comes from these four runs.

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1. Abstract

Two particles can be prepared in an *entangled* state, so that their measurement results are linked no matter how the two are measured. In 1964 John Bell showed that if the world were governed by ordinary local realism — each particle carrying its own predetermined answers — then the correlations between two such measurements would have to obey a strict limit. The CHSH inequality expresses that limit as a single number: any local-realistic theory must satisfy **S ≤ 2**. Quantum mechanics predicts that entangled particles can break this limit, up to a maximum of **2√2 ≈ 2.828**.

In this experiment I prepared the entangled Bell state on the Tuna-17 quantum processor, measured it in the four settings required by the CHSH test, and combined the four measured correlations into S. The experiment returned

$$S = 2.585 \pm 0.017$$

This is clearly above the classical limit of 2 and below the quantum maximum of 2.828, exactly as quantum mechanics predicts for a real, slightly noisy device. The classical bound is exceeded by about **34 standard deviations**, so the violation is not a statistical accident.

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2. Objective

The goal of this experiment is simple to state:

> **Measure the CHSH quantity S on real hardware and check whether it exceeds the classical limit of 2.**

If S stays at or below 2, the results could be explained by a local-realistic model. If S rises above 2, they cannot — the correlations are genuinely quantum. The purpose of the notebook is not to claim a new discovery, but to carry out this well-known test carefully and to document it so that anyone can reproduce it.

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3. Theory

3.1 The Bell state

The experiment begins by preparing two qubits in the entangled Bell state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big).$$

In this state each qubit on its own looks completely random, but the two are perfectly linked: whenever they are measured the same way, they always agree.

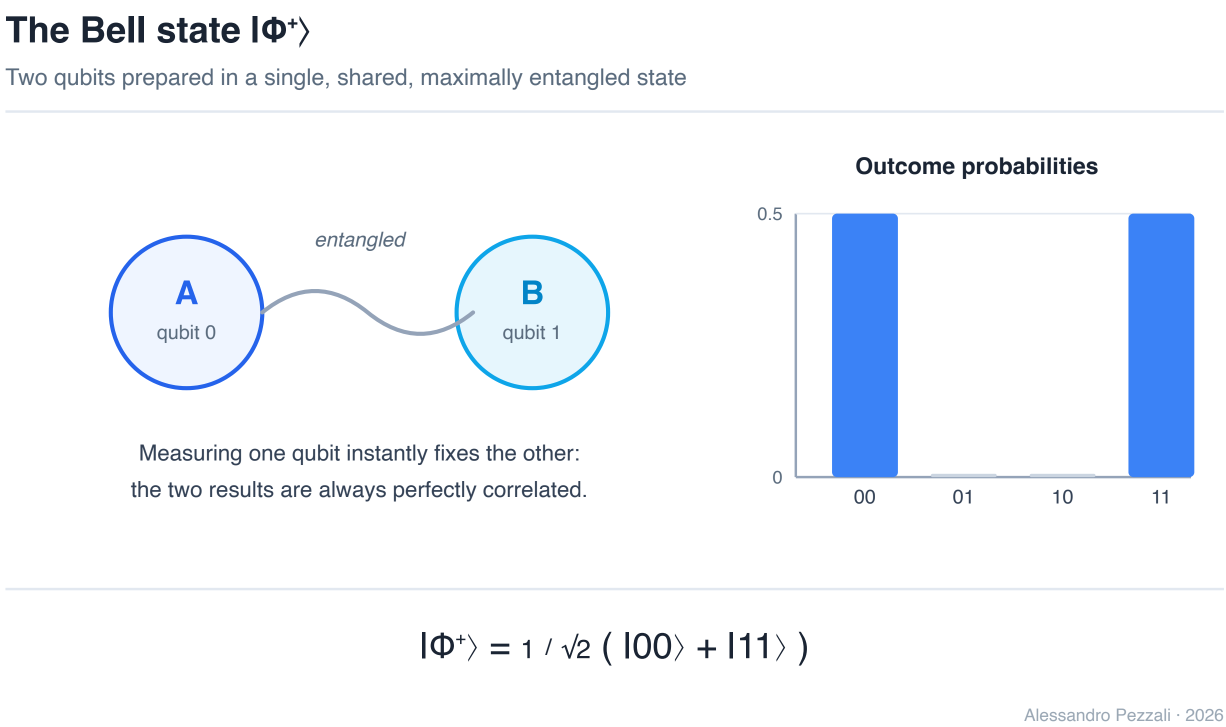

Figure 1. The Bell state $|\Phi^+\rangle$. Only the outcomes 00 and 11 have weight, each with probability ½; 01 and 10 never occur. This perfect agreement is the raw material the CHSH test builds on.

3.2 The CHSH inequality

Alice measures her qubit along one of two directions, a or a'. Bob measures his along one of two directions, b or b'. For each pair of settings we record the **correlation**

$$E = P_{00} + P_{11} - P_{01} - P_{10},$$

which runs from +1 (results always agree) to −1 (results always disagree). The CHSH quantity combines the four correlations:

$$S = E(a,b) + E(a,b') + E(a',b) - E(a',b').$$

Any local-realistic theory obeys **|S| ≤ 2**. Quantum mechanics allows S to reach **2√2 ≈ 2.828**, the *Tsirelson bound*, and no higher.

4. Experimental Design

Why these four particular measurements?

The CHSH quantity is built from exactly four correlations, so the experiment needs exactly four runs — one for each combination of Alice's and Bob's settings. The design question is which measurement directions to choose. The choice matters, because S only reaches its quantum maximum for a specific geometry.

For the Bell state, the correlation between two measurement directions depends only on the angle between them: E ≈ cos(angle). To push S as high as possible, Bob's two directions are placed symmetrically on either side of Alice's Z direction, each 45° from it. With this arrangement, **three of the four Alice–Bob pairs are 45° apart** — giving large positive correlations (cos 45° ≈ +0.71) — while **the fourth pair, (a′, b′), is 135° apart**, giving a large negative correlation (cos 135° ≈ −0.71). That "three positive, one negative" pattern is exactly what the CHSH formula rewards, and it drives S toward its maximum.

The directions I used are:

Party	Setting	Direction	Realised in the circuit by
Alice	*a*	Z axis	measure directly
Alice	*a′*	X axis	apply **H** before measuring
Bob	*b*	(Z + X)/√2	apply **Ry(−π/4)** before measuring
Bob	*b′*	(Z − X)/√2	apply **Ry(+π/4)** before measuring

Table 1. Alice's and Bob's measurement settings and how each is realised in the circuit.

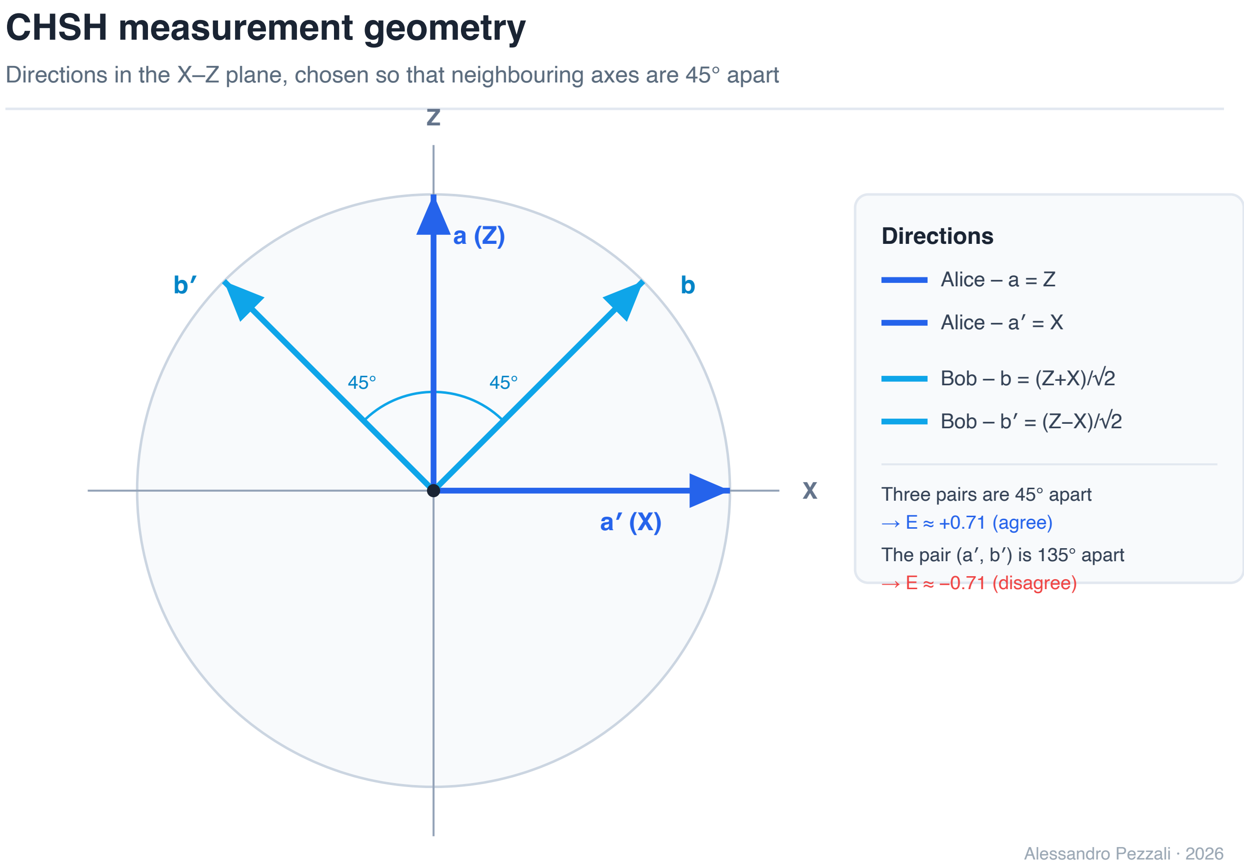

Figure 2. The four measurement directions, drawn in the X–Z plane. Alice's two settings (blue) are 90° apart; Bob's two settings (light blue) sit symmetrically on either side of Alice's Z axis. Three of the four Alice–Bob pairs are then 45° apart (positive correlation) and the remaining pair, (a′, b′), is 135° apart (negative correlation) — the arrangement that maximises S.

Crossing Alice's two settings with Bob's two settings gives the **four configurations** that make up the experiment:

Run	Alice	Bob	Purpose
CHSH_00	Z	$(Z + X)/\sqrt{2}$	correlation E(a, b)
CHSH_01	Z	$(Z - X)/\sqrt{2}$	correlation E(a, b′)
CHSH_10	X	$(Z + X)/\sqrt{2}$	correlation E(a′, b)
CHSH_11	X	$(Z - X)/\sqrt{2}$	correlation E(a′, b′)

Table 2. The four measurement configurations, one per CHSH correlation.

Each configuration uses the *same* entangled state; only the final basis rotations change. This keeps the comparison clean: any difference between runs comes from the measurement setting, not from a different starting state.

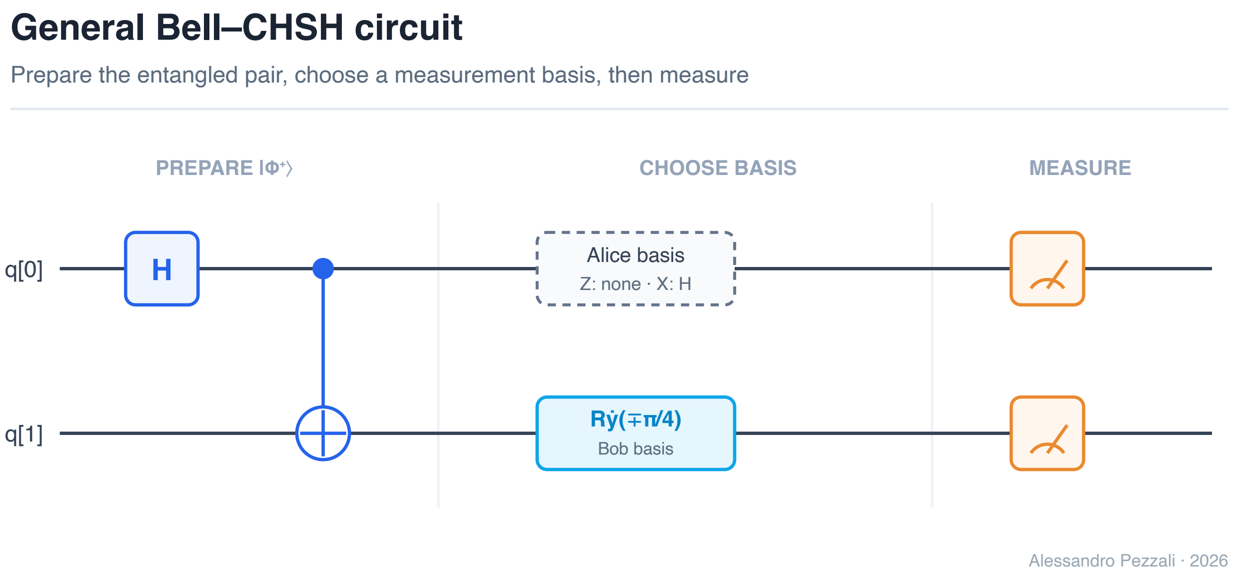

Figure 3. The general circuit template. The first two gates (**H** then **CNOT**) prepare the Bell state and are identical in all four runs. The middle stage sets the measurement basis — nothing or **H** on Alice's qubit, and **Ry(∓π/4)** on Bob's. The last stage measures both qubits.

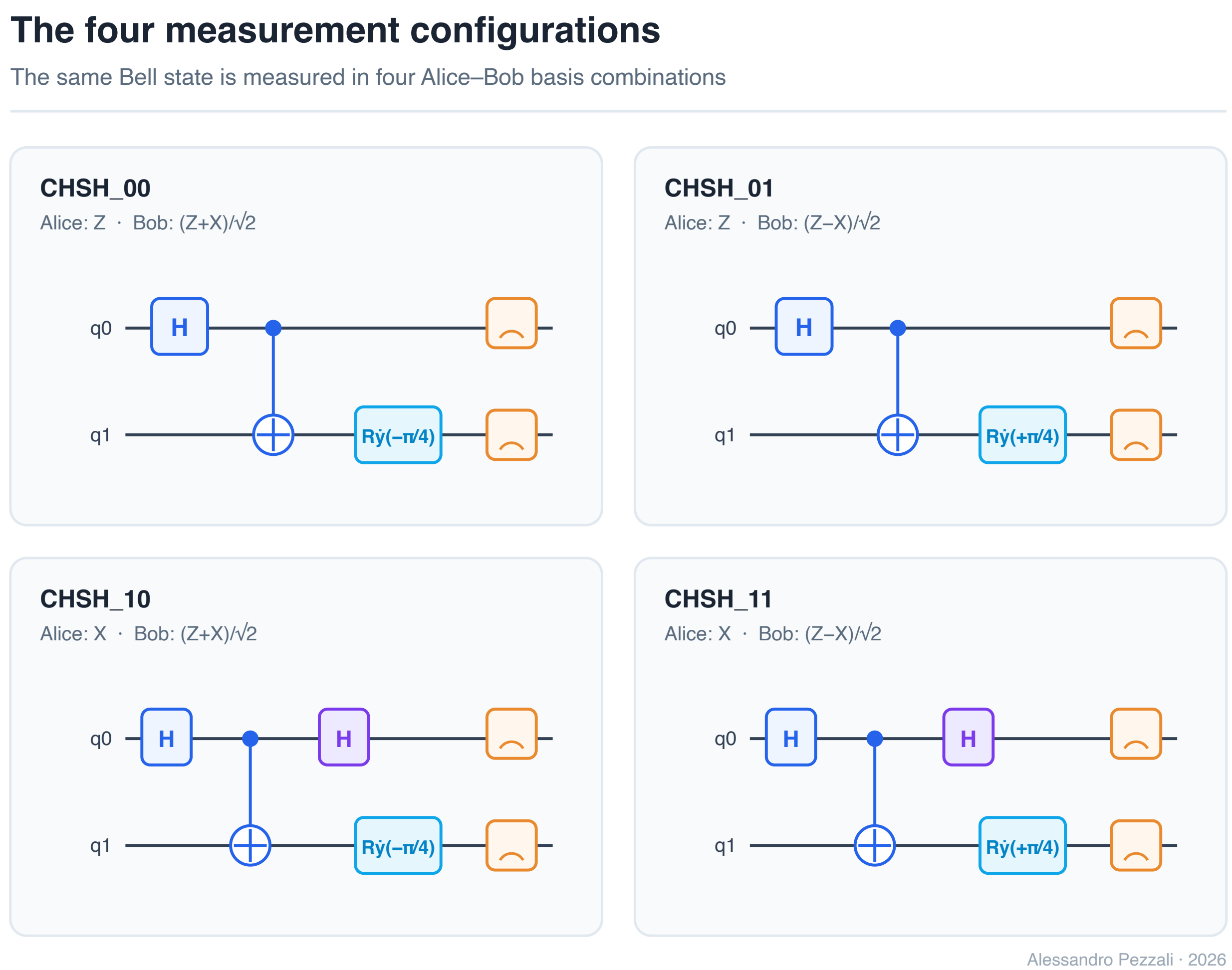

Figure 4. The four programs actually run on the hardware. The preparation block is shared; the small differences in the basis stage (Alice's extra **H** in the bottom row, the sign of Bob's **Ry**) are the entire

experiment.

5. Methods

Hardware and software. All four programs were written in cQASM 3.0 and executed on the Quantum Inspire platform using the **Tuna-17** backend. Each program used two qubits and two classical bits.

State preparation. Every run starts with the same two gates:

```
H q[0]
CNOT q[0], q[1]
```

The Hadamard gate puts qubit 0 into an equal superposition; the CNOT then entangles the two qubits into the Bell state $|\Phi^+\rangle$.

Measurement settings. After preparation, each run applies the basis rotations for its configuration (Section 4) and then measures both qubits:

```
b = measure q
```

Statistics. Each configuration was executed with **8192 shots**. The platform reports the outcome probabilities for the four results 00, 01, 10 and 11.

Session log. The four runs were executed one after another in a single session:

Run	Job ID	Result ID	Timestamp	Duration	Program hash
CHSH_00	1145119	1110686	18/07/2026 17:28:39	4.2858 s	1158991@ce73b1df
CHSH_01	1145125	1110692	18/07/2026 17:29:20	4.3931 s	1158999@63567bd1
CHSH_10	1145132	1110700	18/07/2026 17:30:12	4.2643 s	1159006@8c1c4809
CHSH_11	1145143	1110711	18/07/2026 17:30:53	4.3470 s	1159015@37f3544b

Table 3. Session log for the four runs, as reported by the platform.

The original platform screenshots for all four runs are reproduced in Appendix B.

6. Raw Data

The platform returned the outcome probabilities below (8192 shots per run).

Run	Alice	Bob	P(00)	P(01)	P(10)	P(11)
CHSH_00	Z	$(Z + X)/\sqrt{2}$	0.407	0.088	0.094	0.412
CHSH_01	Z	$(Z - X)/\sqrt{2}$	0.405	0.087	0.092	0.417
CHSH_10	X	$(Z + X)/\sqrt{2}$	0.411	0.083	0.086	0.420
CHSH_11	X	$(Z - X)/\sqrt{2}$	0.090	0.404	0.418	0.089

Table 4. Measured outcome probabilities for the four runs.

Approximate shot counts, reconstructed as round(probability × 8192), are shown only as a guide; they need not sum to exactly 8192 because the displayed probabilities are rounded to three decimals.

Run	00	01	10	11
CHSH_00	3334	721	770	3375
CHSH_01	3318	713	754	3416
CHSH_10	3367	680	705	3441
CHSH_11	737	3310	3424	729

Table 5. Reconstructed approximate shot counts (≈ probability × 8192).

Measured outcome distributions

8192 shots per configuration · Quantum Inspire, Tuna-17 backend

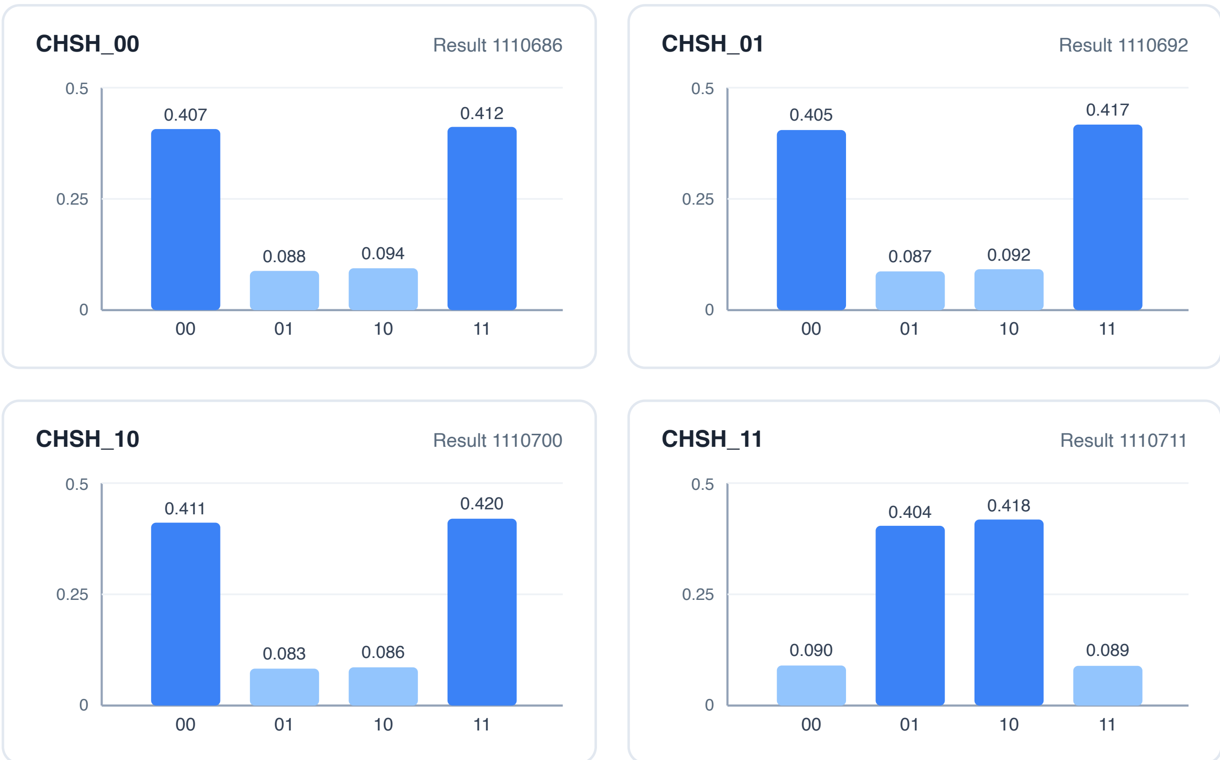

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Figure 5. The measured probability distributions for the four runs. In the first three the outcomes 00 and 11 dominate (results tend to agree); in CHSH_11 the pattern flips to 01 and 10 (results tend to disagree). That single sign flip is what produces the CHSH violation.

7. Analysis

7.1 Correlations

Using $E = P_{00} + P_{11} - P_{01} - P_{10}$ for each run:

Run	Correlation	Value
CHSH_00	E(a, b)	0.407 + 0.412 – 0.088 – 0.094 = **+0.637**
CHSH_01	E(a, b′)	0.405 + 0.417 – 0.087 – 0.092 = **+0.643**
CHSH_10	E(a′, b)	0.411 + 0.420 – 0.083 – 0.086 = **+0.662**
CHSH_11	E(a′, b′)	0.090 + 0.089 – 0.404 – 0.418 = **–0.643**

Table 6. The four measured correlations.

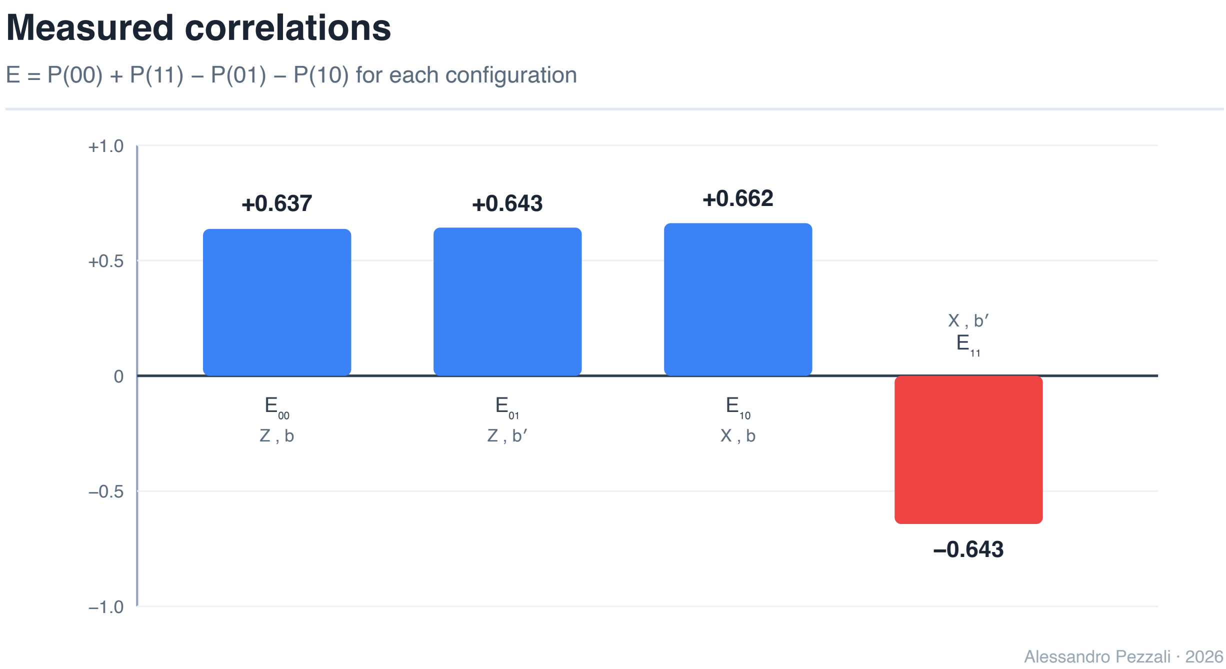

Figure 6. The four measured correlations. Three are strongly positive and one is strongly negative — the sign pattern the CHSH combination is designed to detect.

7.2 The CHSH quantity

$$S = E(a,b) + E(a,b') + E(a',b) - E(a',b')$$

$$S = 0.637 + 0.643 + 0.662 - (-0.643) = 2.585$$

7.3 Uncertainty

Each correlation is estimated from 8192 shots. The statistical uncertainty of a single correlation is approximately σ_E = √((1 − E²)/N), giving about ±0.008 for each run. Combining the four in quadrature:

$$\sigma_S = \sqrt{\sigma_{00}^2 + \sigma_{01}^2 + \sigma_{10}^2 + \sigma_{11}^2} \approx 0.017.$$

So the final result is

$$\boxed{S = 2.585 \pm 0.017.}$$

8. Results

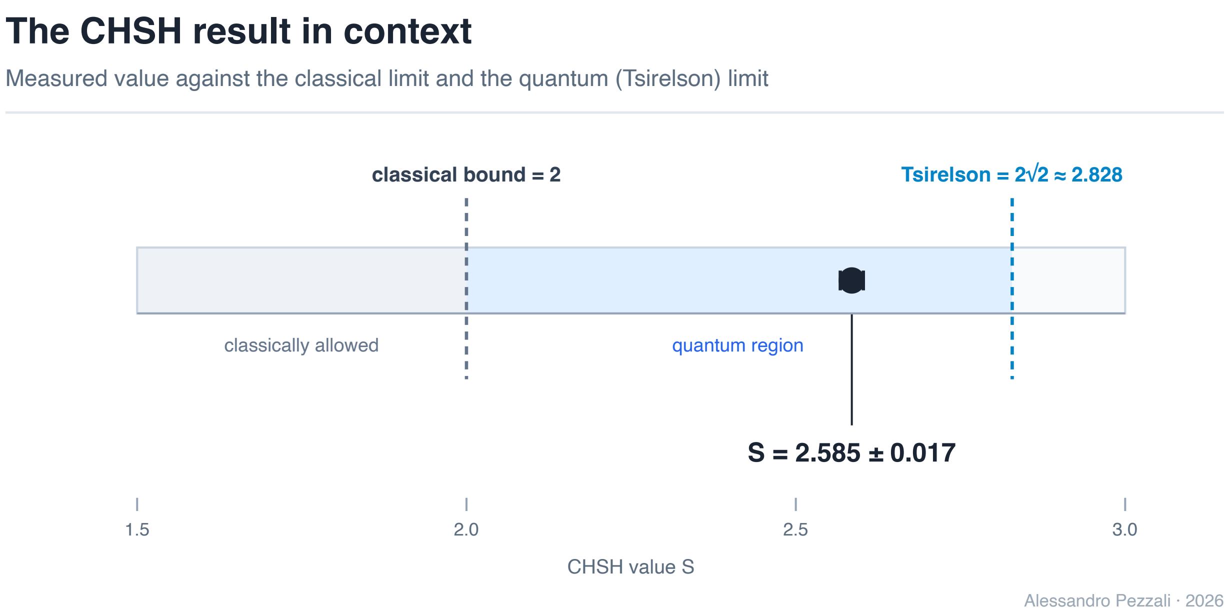

Figure 7. The measured value sits clearly inside the quantum region — above the classical bound of 2 and below the Tsirelson bound of 2√2 ≈ 2.828.

The measured value exceeds the classical limit by

$$\frac{2.585 - 2}{0.017} \approx 34 \text{ standard deviations.}$$

A local-realistic explanation of these four runs is therefore excluded with a very large margin. At the same time S stays safely below the quantum maximum of 2.828, which is exactly what is expected from a real device.

9. Discussion

The result behaves as quantum mechanics predicts, with one honest caveat: the measured S of 2.585 is a little below the ideal 2.828. The ratio

$$\frac{2.585}{2.828} \approx 0.91$$

can be read as the **visibility** of the entangled state — how close the hardware came to a perfect Bell pair. A value near 0.91 is normal for present-day quantum processors, where gate imperfections, decoherence and readout errors slightly weaken the correlations.

Two features give confidence that the effect is real rather than an artefact:

1. **Consistency across runs.** The three "agreeing" correlations (0.637, 0.643, 0.662) are close to one another, and the "disagreeing" one (−0.643) is their mirror image. This is the clean, symmetric pattern the ideal experiment predicts.
2. **Large margin.** The classical bound is exceeded by roughly 34 standard deviations, far beyond anything statistical noise could produce over 8192 shots per setting.

No claim is made here beyond what the data support: on this hardware, in this session, the CHSH inequality was violated.

Scope and limits. This notebook documents four runs from a single session. It does not close the well-known loopholes of a rigorous Bell test (for example the locality loophole), and no independent device-calibration data were recorded beyond the metadata reported by the platform. The result should be read as a faithful, reproducible demonstration, not as a loophole-free Bell experiment.

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10. Conclusion

I prepared an entangled Bell state on the Tuna-17 quantum processor and measured it in the four settings of the CHSH test. Combining the four measured correlations gave

$$S = 2.585 \pm 0.017,$$

which exceeds the classical limit of 2 by about 34 standard deviations and stays below the quantum maximum of 2.828. The experiment is a clean, reproducible demonstration that the correlations produced by this hardware cannot be explained by local realism.

Appendix A — Source Code

All four programs are written in cQASM 3.0. The preparation block is identical in every run; only the measurement stage changes. The code below is reproduced exactly as executed; only the inline comments have been translated from the original Italian shown in the Appendix B screenshots (for example *"Bob misura lungo (Z + X) / sqrt(2)"* → *"Bob measures along (Z + X) / sqrt(2)"*). The executable instructions are unchanged.

A.1 CHSH_00 — Alice Z, Bob (Z + X)/√2

```
version 3.0

qubit[2] q
bit[2] b

H q[0]
CNOT q[0], q[1]

// Bob measures along (Z + X) / sqrt(2)
Ry(-0.7853981634) q[1]

b = measure q
```

A.2 CHSH_01 — Alice Z, Bob $(Z - X)/\sqrt{2}$

```
version 3.0

qubit[2] q
bit[2] b

H q[0]
CNOT q[0], q[1]

// Bob measures along (Z - X) / sqrt(2)
Ry(0.7853981634) q[1]

b = measure q
```

A.3 CHSH_10 — Alice X, Bob $(Z + X)/\sqrt{2}$

```
version 3.0

qubit[2] q
bit[2] b

H q[0]
CNOT q[0], q[1]

// Alice measures in the X basis
H q[0]

// Bob measures along (Z + X) / sqrt(2)
Ry(-0.7853981634) q[1]

b = measure q
```

A.4 CHSH_11 — Alice X, Bob $(Z - X)/\sqrt{2}$

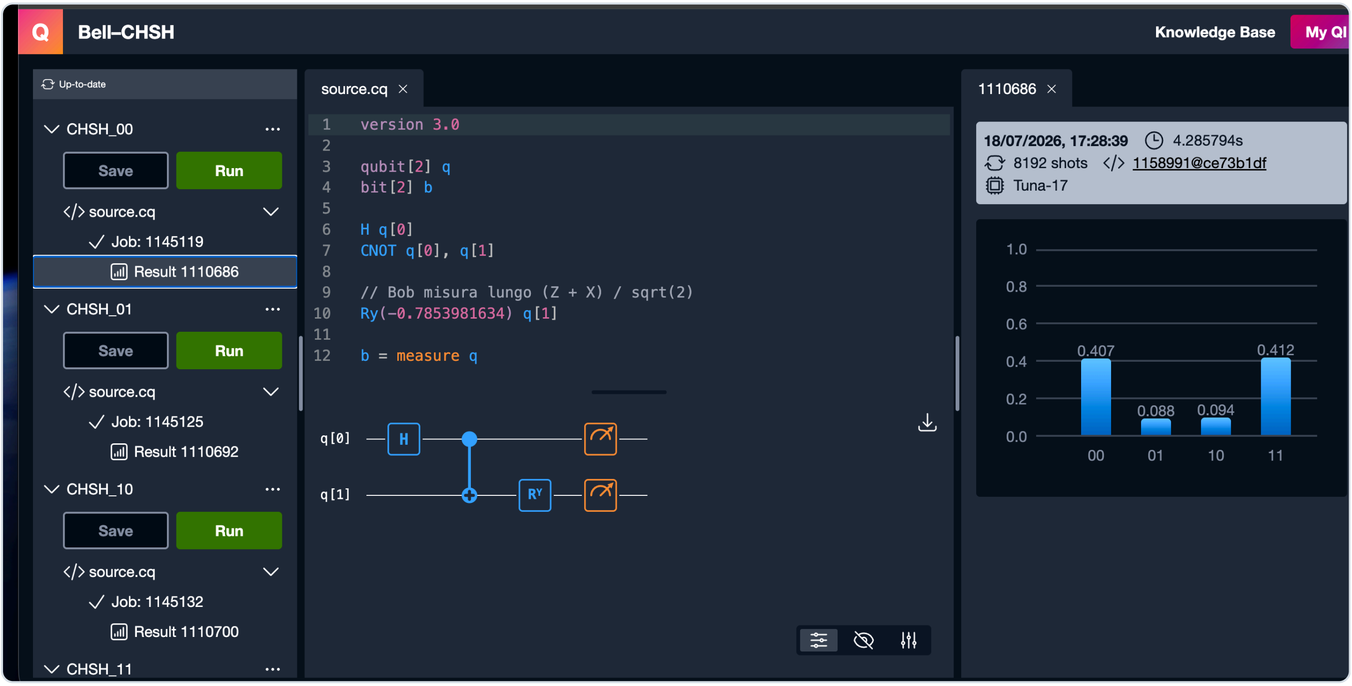
```
version 3.0

qubit[2] q
bit[2] b

H q[0]
CNOT q[0], q[1]

// Alice measures in the X basis
H q[0]

// Bob measures along (Z − X) / sqrt(2)
Ry(0.7853981634) q[1]

b = measure q
```

> Note: Ry(0.7853981634) = Ry($\pi/4$). The angle $\pm\pi/4$ places Bob's measurement halfway between the Z and X axes, giving the $(Z \pm X)/\sqrt{2}$ directions of Section 4.

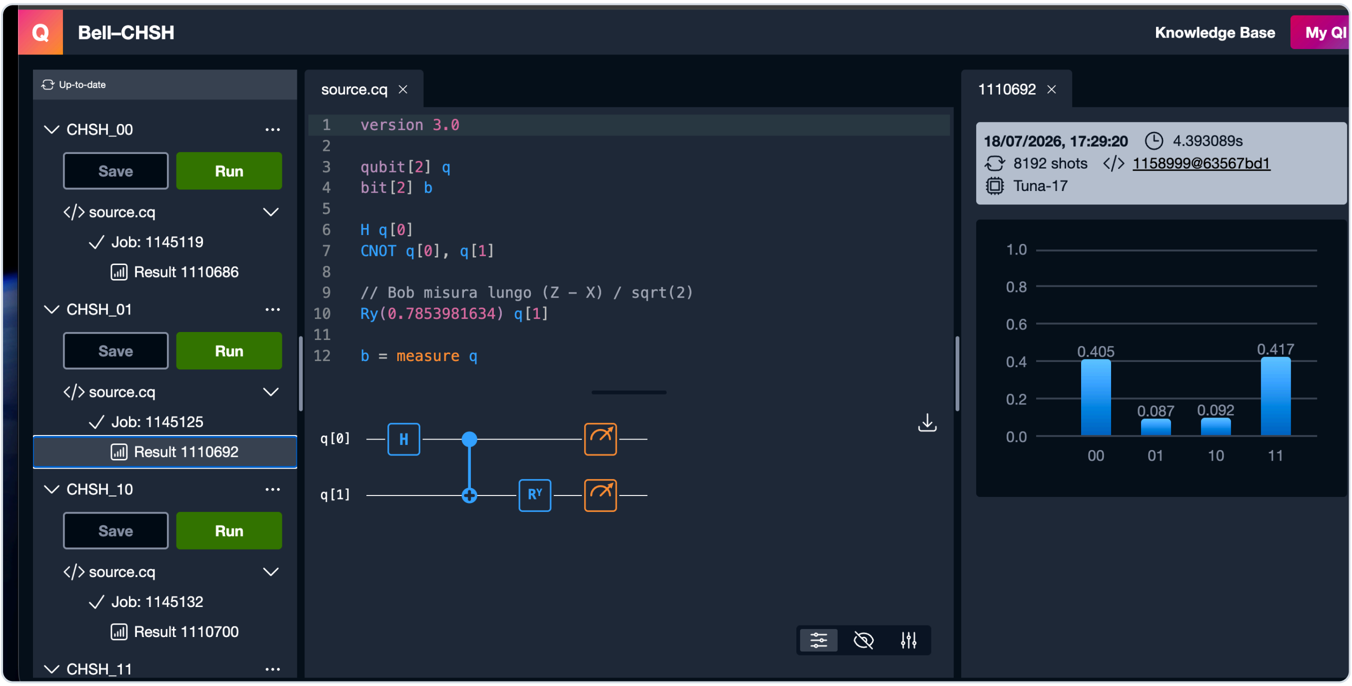
Appendix B — Original Platform Screenshots

The original Quantum Inspire result screens for the four runs, kept for provenance. Each shows the executed program, the circuit, the backend (Tuna-17), the shot count (8192), the timestamp, and the measured outcome probabilities.

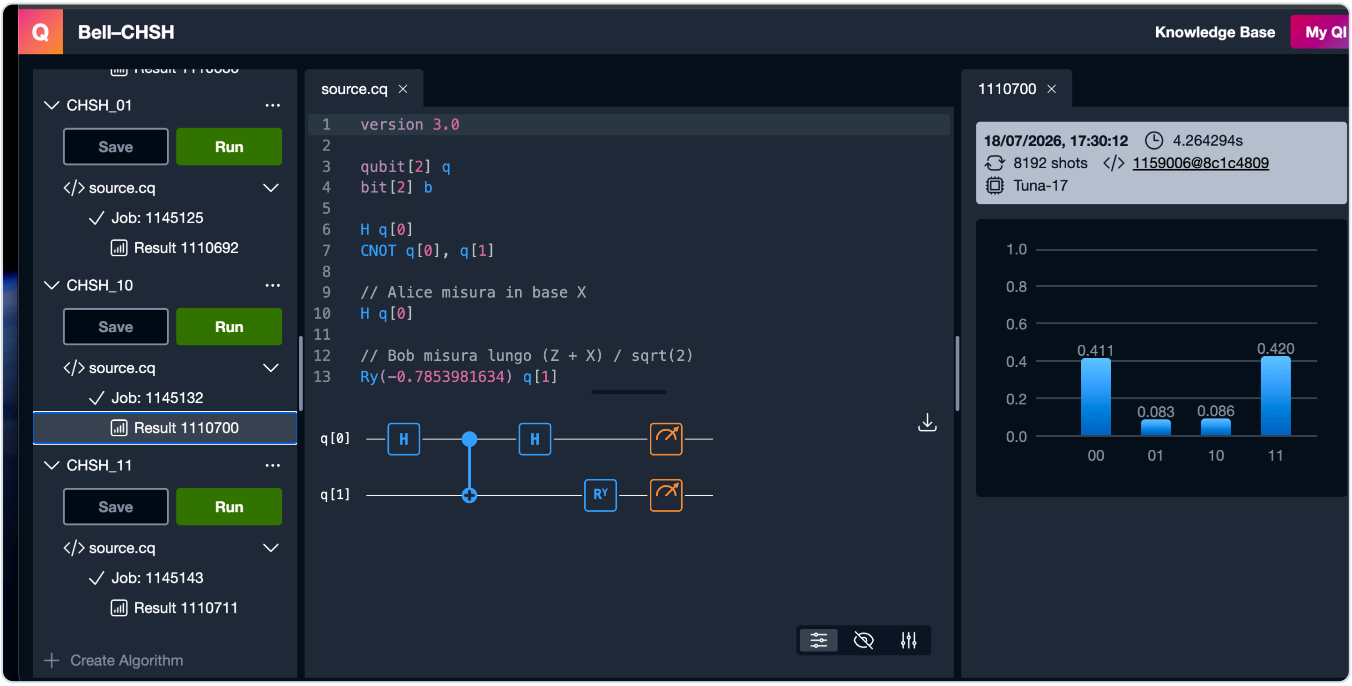
CHSH_00 — Result 1110686



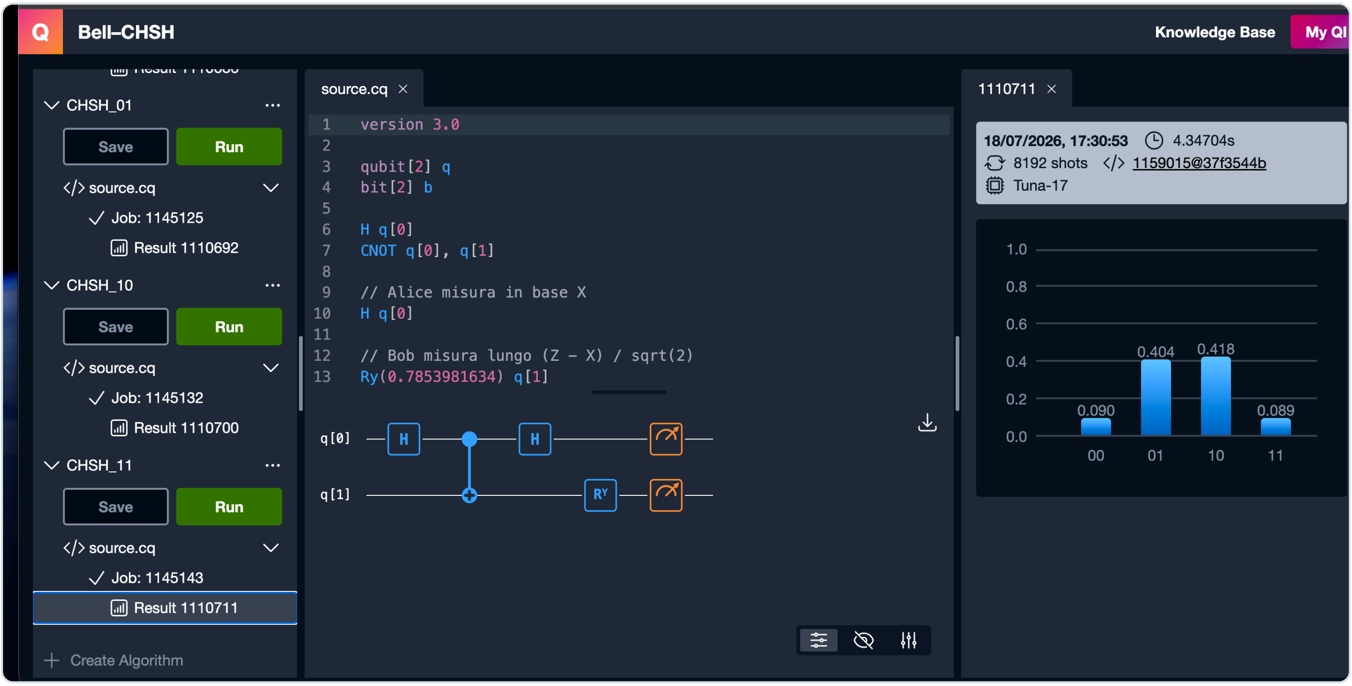
CHSH_01 — Result 1110692



CHSH_10 — Result 1110700



CHSH_11 — Result 1110711



References

1. J. S. Bell, *On the Einstein Podolsky Rosen paradox*, Physics **1**, 195 (1964).
2. J. F. Clauser, M. A. Horne, A. Shimony, R. A. Holt, *Proposed experiment to test local hidden-variable theories*, Physical Review Letters **23**, 880 (1969).
3. B. S. Cirel'son (Tsirelson), *Quantum generalizations of Bell's inequality*, Letters in Mathematical Physics **4**, 93 (1980).
4. Quantum Inspire — quantum computing platform documentation (Tuna-17 backend).

Sole author: Alessandro Pezzali. Independent personal research and educational project. Quantum Inspire and the Tuna-17 backend are used solely as the experimental platform on which the quantum circuits were executed.